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Almost S_g- Continuity in Topological Spaces

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Abstract

In the present paper, we introduce a new class of functions in between topological spaces, namely the class of almost s_g-continuous functions. Characterizations and basic properties of this class of functions are obtained as well as the relationships of it with some other classes of functions are studied.

I. Introduction and Preliminaries

The notion of semi-open sets and generalized closed (briefly, g-closed) sets introduced by Levine in 1963 [18] and 1970 [17] and presented fundamental results for these sets, respectively. So many mathematicians generalized many sets, and then they used those sets to generalize some types of continuity, Also, they obtained thier properties and relationships among them. The purpose of this paper is to introduce the notion of an almost S_g-continuous function and investigate some of the properties for this class of functions.

Throughout this paper, (X,τ) and (Y,σ) (or simply; X and Y) are always topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of X. Then clA and $intA$ are denote the closure and interior of A. A subset A is said to be semi-open [18] (resp., preopen [20], α-open [22], β-open [1] or semi-preopen [3], regular open [30] , δ-semi-open [26] and g-closed [17]) sets if and only if $A \subseteq clintA$ (resp. $A \subseteq intclA$, $A \subseteq intclintA$, $A \subseteq clintclA$, $A = intclA$, $A \subseteq clint_{\delta}A$ and $clA \subseteq U$, whenever U is open and $A \subseteq U$). The complement of a semi-open (resp. preopen, α-open, β-open, regular open, δ-semi-open and g-closed) set is said to be semi-closed (resp. preclosed, α-closed, β-closed, regular closed, δ-semi-closed and g-open).The family of all semi-open (resp., preopen, α-open, β-open, regular open, δ-semi-open and g-closed) subsets of a space X are denoted by $SO(X)$ (resp., $PO(X), \alpha O(X), \beta O(X), RO(X), \delta SO(X)$ and $GC(X)$). The family of all regular open [30] subsets of a space (X, τ) form a base for a topology on X, which is denoted by τ_s and it is weaker than τ, and this topology is called semi-regularization of X. A subset A of a space X is called θ-semi-open [16] (resp., δ-open [31] and θ-open [31]) if for each $x \in A$, there exists a semi-open (resp. open, open) set G such that $x \in G \subseteq clG \subseteq A$ (resp., $x \in G \subseteq intclG \subseteq A$ and $x \in G \subseteq clG \subseteq A$). The complement of θ-semi-open (resp., δ-open and θ-open) set is said to be θ-semi-closed (resp. δ-closed and θ-closed). The family of all θ-semi-open (resp., δ-open and θ-open) subsets of a space X are denoted by $\theta SO(X)$ (resp. $\delta O(X)$ and $\theta O(X)$). A subset A of a space X is called P_S-open [5] if for each $x \in A \in PO(X)$, there exists a semi-closed set F such that $x \in F \subseteq A$. Further, a δ-semi-open A of a space X is said to be δs_C-open[7], if for each $x \in A$ there exists a closed set F such that $x \in F \subseteq A$. The family of all P_S-open (resp. δs_C-open) subsets of a space X are denoted by $P_S O(X)$(resp. $\delta s_C O(X)$). A function $f: X \to Y$ is said to be continuous [28] (resp. almost α-continuous [24]almost precontinuous [25], almost β- continuous [4], δs_c-continuous [7]) if the inverse image of each open (resp. regular open, regular open, regular open, open) subset of Y is open (resp. α-open, preopen, β-open, δs_C-open) in X. A function $f: X \to Y$ is called almost P_s-continuous[5] (resp. almost θs-continuous.[2], almost semi-continuous[14]) if for each $x \in X$ and each open set V containing $f(x)$, there exists a P_s-open (resp. semi-open, semi-open) set U in X containing x such that $(f(U) \subseteq intcl(V))$ (resp. $f(clU) \subseteq intclV$, $f(U) \subseteq intclV$). A function $f: X \to Y$ is said to be δ-continuous [23](resp. almost strongly θ-continuous [14], almost continuous in the sense of Singal and Singal [29], almost λ-continuous [15]) if for each $x \in X$ and each open set V of Y containing *f(x)*, there exists an open (resp. open, open, λ-open) set U of X containing x such that $f(intcl(U)) \subseteq int(cl(V))$ (resp. $f(clU) \subseteq scl(V), f(U) \subseteq intcl(V), f(U) \subseteq int(clV)$)).

Definition 1.1 [8] A semi-open set A of a space X is said to be an all $\boldsymbol{S_g}$**-open** set if for each $x \in A$, there exists a g-closed set F such that $x \in F \subseteq A$. The family of S_g–open subsets of X is denoted by $S_g O(X)$.

Definition 1.2 A space X is said to be:

1- extremally disconnected [32] if the closure of every open subset of X is open.

2- locally indiscrete[10] if every open subset of X is closed.

3- hyperconnected [10] if every non-empty open subset of X is dense.

4- s*- regular [21] if for any semi-regular set A and $x \notin A$, there exist disjoint open sets U and V such that $A \subseteq U$ and $x \in V$.

5- semi-regular,[11] if for each semi-closed set A and each point $x \notin A$, there exist disjoint $U, V \in SO(X)$ such that $x \in U$ and $A \subseteq V$.

The intersection of all $\boldsymbol{S_g}$-closed sets of X containing A is called the $\boldsymbol{S_g}$-closure of A and is denoted by $\boldsymbol{S_g}cl(A)$. The union of all $\boldsymbol{S_g}$-open sets of X contained in A is called the $\boldsymbol{S_g}$- interior of A and is denoted by $\boldsymbol{S_g}int(A)$.

Corollary 1.3 [9] If A is any subset of a space X. Then $X \setminus S_g int(A) = S_g cl(X \setminus A)$.

Definition 1.4 [9] A function $f: X \to Y$ is called S_g-continuous at a point $x \in X$, if for each open set U of Y containing $f(x)$ there exists an S_g-open set G in X containing x such that $f(G) \subseteq U$. If f is S_g-continuous at every point x of X , then it is called S_g-continuous.

Proposition 1.5 [9] A function $f: X \to Y$ is S_g-cotinuous if and only if for every open subset U of Y, $f^{-1}(U)$ is an S_g-open set in X.

Definition 1.6 Let X be a space and let $x \in X$.Then a subset N of X is said to be S_g-neighborhood of x if there exists an S_g-open set U in X such that $x \in U \subseteq N$.

Proposition 1.7 A subset A of a space X is an S_g-open set if and only if it is S_g-neighborhood of each of it is points.

Proof. Obvious.

Theorem 1.8 [18] Let A be any subset of a space X. Then $A \in SO(X)$ if and only if $cl(A) = clint(A)$.

Corollary 1.9 [8] Every regular closed subset of a space X is an S_g-open set.

Proposition 1.10 [8] If (X, τ) is a completely regular or a regular space, then $\tau \subseteq S_gO(X)$.

Proposition 1.11 [8] If a space X is locally indiscrete, then $SO(X) = S_gO(X)$.

Corollary 1.12 [8] If a space X is locally indiscrete, then $S_gO(X) = \tau$

Theorem 1.13 The following conditions are equivalent for a space (X, τ):

1- X is extremally disconnected

2- $RO(X) = RC(X)$ [13]

3- Every semi-open subset of X is α-open [19]

Lemma 1.14 [27] Every regular open subset of a space X is δ-open.

Corollay 1.15 [3] A subset A of a space X is β-open if and only if $cl(A)$ is regular closed.

Theorem 1.16 [21] A space X is S*-regular if and only if it is extremally disconnected.

Theorem 1.17 [24] For a function $f: X \to Y$ the following are equivalent:

1- f is almost α- continuous

2- for each $x \in X$ and each $V \in \sigma$ containing $f(x)$, there exists $U \in \tau^{\alpha}$ containing x such that $f(U) \subseteq int(cl(V))$

Theorem 1.18 [5] If a space (X, τ) is locally indiscrete, then $P_sO(X) = \tau$

Lemma 1.19 [6] A space X is locally indiscrete if and only if every λ-open subset of X is open in X .

Corollary 1.20 [5] Every almost P_s- continuous function is almost precontinuous.

Definition 1.21 [12] For a function $f: X \rightarrow Y$, the graph function $g: X \rightarrow X \times Y$ of f is defined by $g(x) = (x, f(x))$ for each $x \in X$.
Theorem 1.22 [14] For a function $f: X \rightarrow Y$, the following are equivalent:
1-f is almost strongly θ- continuous
2-The inverse image of each regular open set in Y is θ-open in X
3-The inverse image of each δ- open set in Y is θ-open in X.
Proposition 1.23 [8] Every θ-open and θ-semi open subset of X are S_g-open.

2. Almost S_g-continuous Functions

In this section, we introduce and investigate the almost s_g –continuity in topological spaces, and then we give its characterizations and relationships with some other types of continuity.

Definition 2.1 A function $f: X \rightarrow Y$ is called **almost s_g-continuous** if for each $x \in X$ and each open set H in Y containing $f(x)$, there exists an S_g-open set G in X containing x such that $f(G) \subseteq intclH$.

The following result follows directly from their definitions:
Lemma 2.2 Every δS_c-continuous and S_g-continuous functions are almost S_g-continuous.

The converse is not true in general as shown in the following example:

Example 2.3 Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$. Thus $SO(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$ and $S_gO(X = \{\emptyset, X, \{a, c\}\}$. Defined a function $f: X \rightarrow X$ by $f(a) = f(b) = b$ and $f(c) = a$, then f is almost S_g-continous but it is neither S_g-continuous nor δS_c-continuous.
Lemma 2.4 Every almost θs-continuous function is almost S_g-continuous

Proof. Let $f: X \rightarrow Y$ be almost θs-ccontinuous, let $x \in X$ and H any open subset of Y containing $f(x)$.Then there exists a semi-open set U in X containing x such that $f(clU) \subseteq intclH$. Since U is semi-open in X, then by Theorem.1.8, $clU = clintU$. This implies that $clU \in RC(X)$, so by Corollary1.9, we have $F = clU \in S_gO(X)$.Therefore, $f(F) \subseteq intclH$, and hence f is almost S_g-continuous function.

The converse of Proposition 2.4 is not true in general as it is shown in the following example:

Example 2.5 $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$, $SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ and $S_gO(X) = \{\emptyset, X, \{b, d\}, \{a, d\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}\}$. Then we define a function $f: X \rightarrow X$ by $f(a) = f(d) = a$, $f(b) = b$ and $f(c) = c$. Therefore, f is almost S_g-continuous, but not almost θs-continuous since $a \in X$ and for $\{a\} \in \tau$ there does not exists a semi-open set G in X containing a such that $f(clG) \subseteq intcl(\{a\})$.

Since every S_g-open subset of X is semi-open, we get the following result:

Lemma 2.6 Every almost S_g-continuous function is almost semi-continuous.

The converse of Lemma 2.6 is not true in general as we have shown in the following example:

Example 2.7 Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a, b\}, \{c\}, \{a, b, c\}\}$. Then $O(X) = \{\emptyset, X, \{c\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}\}$, $GC(X) = \{\emptyset, X, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}$, $\{ a, c, d\}\{b, c, d\}\}$ and $S_gO(X) = \{\emptyset, X, \{c, d\}, \{a, b, d\}\}$. The identity function I is almost semi-continuous but not almost S_g-continuous, since for $c \in X$ and for $\{c\} \in \tau$ but there is no an S_g-open set G in X containing c such that $I(G) \subseteq intcl(\{c\}) = \{c\}$.

Theorem 2.8 Let $f: X \to Y$ be a function. If for each $x \in X$ and each open set H in Y containing $f(x)$, there exists a semi-open set U in X containing x such that $f(clU) \subseteq$ intcl$H$, then f is an almost S_g-continuous function.

Proof. Let for each $x \in X$ and each open set H in Y containing $f(x)$, there exists a semi-open set U in X containing x such that $f(clU) \subseteq intclH$. Put $G = clU$, then G is regular closed set in X, thus by Corollary 1.9, G is an S_g-open set containing x such that $f(G) \subseteq intclH$. Thus, f is an almost S_g-continuous function.

The converse of Theorem 2.8, is not true in general as it is shown in Example 2.5, f is almost S_g-continuous, but for $a \in X$ and for $\{a\} \in \tau$ there is no semi-open set G in X containing a such that $f(clG) \subseteq intcl(\{a\})$.

Theorem 2.9 For a function$f: X \to Y$, the following statements are equivalent:

1- f is almost S_g-continuous.

2- For each $x \in X$ and each regular open set V of Y containing $f(x)$, there exists an S_g-open set U in X containing x such that $f(U) \subseteq V$.

3- For each $x \in X$ and each δ-open set V of Y containing $f(x)$, there exists an S_g-open set U in X containing x such that$f(U) \subseteq V$.

Proof. (1) $\longrightarrow$ (2) Let $x \in X$ and V be any regular open subset of Y such that $f(x) \in V$. Since every regular open set is open and since f is almost S_g-continuous, then there exists an S_g-open set G such that $x \in G$ and $(G) \subseteq intclV$, but since $intclV$ =V, so $f(G) \subseteq V$.

(2) $\longrightarrow$ (3) Let $x \in X$ and V be any δ-open set of Y such that $f(x) \in V$. Then, there exists an open set H containing $f(x)$ such that $H \subseteq intclH \subseteq V$. Since $intclH$ is a regular open subset of Y containing $f(x)$. So by (2), there exists an S_g-open set U in X containing x such that $f(U) \subseteq intclH$. Thus, $f(U) \subseteq V$.

(3) $\longrightarrow$ (1) Let $x \in X$ and V be any open set of Y such that $f(x) \in V$. Then $intclG$ is a δ-open set, therefore, by (3) there exists an S_g-open set U in X containing x such that $(U) \subseteq intclG$. This implies that f is an almost S_g-continuous function.

Theorem 2.10 A function $f: X \to Y$ is almost S_g-continuous if and only if the inverse image of each regular open set in Y is an S_g-open set in X.

Proof. It is Obvious.

Corollary 2.11 A function $f: X \rightarrow Y$ is almost S_g-continuous if and only if the inverse image of each regular closed set in Y is an S_g-closed set in X.

Proposition 2.12 A function $f: X \rightarrow Y$ is almost S_g-continuous if and only if the inverse image of each δ-open set in Y is an S_g-open set in X.

Proof. It is obvious.

Corollary 2.13 A function $f: X \rightarrow Y$ is almost S_g-continuous if and only if the inverse image of each δ-closed set in Y is an S_g-closed set in X.

Proposition 2.14 Let $f: X \rightarrow Y$ be a function. If the inverse image of every semi-open set in Y is a regular closed set in X, then f is almost S_g-continuous function.

Proof. Suppose the inverse image of every semi-open set in Y be a regular closed set in X. Let $x \in X$ and H be an open subset of Y containing $f(x)$. Then $f^{-1}(H)$ is a regular closed set in X containing x. Thus, $f^{-1}(H)$ is an S_g-open set in X containing x. Since $f(f^{-1}(H)) \subseteq H$, then f is an almost S_g-continuous function.

The converse of Proposition 2.14 is not true in general. Since the function f in Example 2.3 is almost S_g-continuous but $\{a\}$ is semi-open and $f^{-1}(\{a\}) = \{b\}$ is not regular closed.

Corollary 2.15 Let $f: X \rightarrow Y$ be a function. If the inverse image of every semi-open set in Y is θ-semi-open set in X, then f is almost S_g-continuous.

Theorem 2.16 A function $f: X \rightarrow Y$ is almost S_g-continuous if and only if for each open set V of Y, $f^{-1}(V) \subseteq S_g int(f^{-1}(int(clV))$.

Proof. Let $f: X \rightarrow Y$ be almost S_g-continuous and let V be any open subset of Y. Let $x \in f^{-1}(V)$. Then$f(x) \in V$. Since f is almost S_g-continuous, there exists an S_g-open set U in X containing x such that $f(U) \subseteq intclV$. This implies that $x \in U \subseteq f^{-1}(intclV)$. Therefore, $x \in S_g int(f^{-1}(int(clV))$. Further, $f^{-1}(V) \subseteq S_g int(f^{-1}(int(clV))$.

Conversely, suppose for each open set V of Y, $f^{-1}(V) \subseteq S_g int(f^{-1}(int(clV))$. To show f is almost S_g-continuous. Let $x \in X$ and H be any open set in Y containing $f(x)$. So by hypothesis $f^{-1}(H) \subseteq S_g int(f^{-1}(int(clH))$. This implies that $x \in S_g int(f^{-1}(int(clH) \subseteq f^{-1}(int(clH))$. Set $U = S_g int(f^{-1}(int(clH))$ which is an S_g-open set in X containing x such that $f(U) \subseteq intclH$. Thus, f is almost S_g-continuous.

Theorem 2.17 For a function $f: X \rightarrow Y$, the following statements are equivalents:

1- f is almost S_g-continuous .

2- $f(S_g cl(A)) \subseteq cl_\delta f(A))$, for each subset A of X.

3- $S_g cl(f^{-1}(B)) \subseteq f^{-1}(cl_\delta B)$, for each subset B of Y.

4- $f^{-1}(int_\delta B) \subseteq S_g int(f^{-1}(B))$, for each subset B of Y.

5- $S_g cl(f^{-1}(B)) \subseteq f^{-1}(clB)$, for each semi-open set B in Y.

Proof. (1) ⟶(2) Let A be any subset of X. Then $cl_\delta f(A))$ is δ-closed in Y. Since f is almost S_g-continuous, then by Corollary 2.13, $f^{-1}(cl_\delta f(A))$ is S_g-closed in X. So $S_g cl\, f^{-1}(cl_\delta f(A)) = f^{-1}(cl_\delta f(A))$. Now, since $f(A) \subseteq cl_\delta f(A)$, this implies that $A \subseteq f^{-1}(cl_\delta f(A))$. Thus $S_g cl(A) \subseteq S_g cl(f^{-1}(cl_\delta f(A))$, so we have $S_g cl(A) \subseteq f^{-1}(cl_\delta f(A))$. Hence $f(S_g cl(A)) \subseteq (cl_\delta f(A))$.

(2) ⟶(3) Let B be any subset of Y. Then $f^{-1}(B) \subseteq X$. Therefore, by (2) we have

$f(S_g cl(f^{-1}(B))) \subseteq cl_\delta f(f^{-1}(B))) \subseteq cl_\delta B$. Then $f^{-1}(f(S_g cl(f^{-1}(B)))) \subseteq f^{-1}(cl_\delta B)$. Hence $S_g cl(f^{-1}(B)) \subseteq f^{-1}(cl_\delta B)$.

(3) ⟶(4) Let B be any subset of Y. Then $Y \backslash B$ is a subset of Y, so by (3) ,we have

$S_g cl(f^{-1}(Y \backslash B)) \subseteq f^{-1}(cl_\delta(Y \backslash B))$. Thus $S_g cl(X \backslash f^{-1}(B)) \subseteq f^{-1}(Y \backslash int_\delta B)$, by Corollary 1.3, $X \backslash S_g int(f^{-1}(B)) \subseteq (X \backslash f^{-1} int_\delta B)$. Hence $f^{-1}(int_\delta B) \subseteq S_g int(f^{-1}(B)$

(4) ⟶(5) Let B be any semi-open subset of Y. Then by Theorem1.8, clB is regular closed in Y. Therefore, $Y \backslash clB$ is regular open in Y. Thus by Lemma1.14, $Y \backslash clB$ is δ-open in Y, then by (4) we have $f^{-1}(Y \backslash clB) \subseteq S_g int(f^{-1}(Y \backslash clB))$. So $f^{-1}(Y \backslash clB)$ is an S_g-open set, this implies that $X \backslash f^{-1}(clB)$ is an S_g-open set in X, that is $f^{-1}(clB)$ is S_g-closed. But $f^{-1}(B) \subseteq f^{-1}(clB)$. Hence $S_g cl f^{-1}(B) \subseteq f^{-1}(clB)$.

(5) ⟶ (1) Let B be any regular open subset of Y. Then $Y \backslash B$ is a regular closed set in Y. This implies that $cl(Y \backslash B) = clint(Y \backslash B)$, then by Theorem1.8, $Y \backslash B$ is semi-open subset of Y. Hence by (5) $S_g cl f^{-1}(Y \backslash B) \subseteq f^{-1}(cl(Y \backslash B)) = f^{-1}(Y \backslash B)$, therefore $f^{-1}(Y \backslash B)$ is S_g-closed in X, that is $X \backslash f^{-1}(B)$ is S_g-closed. Then $f^{-1}(B)$ is an S_g-open set. Thus by Theorem 2.10, f is almost S_g-continuous.

Proposition 2.18 A bijective function $f: X \to Y$ is almost S_g-continuous if and only if

$int_\delta(f(A)) \subseteq f(S_g intA)$, for each subset A of X.

Proof. Let A be any subset of X. Then $f(A)$ is a subset of Y. Since f is almost S_g-continuous, then by Theorem 2.17, $f^{-1}(int_\delta f(A)) \subseteq S_g int(f^{-1}(f(A))$. This implies that $int_\delta f(A)) \subseteq f(S_g int(A))$.

Conversely, let $x \in X$ and V be any regular open subset of Y containing $f(x)$. Then $x \in f^{-1}(V)$ and $f^{-1}(V)$ is a subset of X. By hypothesis, we have $int_\delta f(f^{-1}(V)) \subseteq f(S_g int f^{-1}(V))$, then $int_\delta (V) \subseteq f(S_g int f^{-1}(V))$. Since V is a regular open set, then by Lemma1.14, V is δ-open. Hence $V \subseteq f(S_g int f^{-1}(V))$, so $f^{-1}(V) \subseteq (S_g int f^{-1}(V))$. Thus, $f^{-1}(V)$ is an S_g-open set in X which containing x. Hence by Theorem2.16, f is almost S_g-continuous.

Theorem 2.19 For a function$f: X \to Y$, the following statements are equivalent:

1- f is almost S_g-continuous .

2- $S_g clf^{-1}(V) \subseteq f^{-1}(clV)$, for each β-open set V of Y.

3- $f^{-1}(intF) \subseteq (S_g intf^{-1}(F))$, for each β-closed set F of Y.

4- $f^{-1}(intF) \subseteq (S_g intf^{-1}(F))$, for each semi-closed set F of Y.

5- $S_g clf^{-1}(V) \subseteq f^{-1}(clV)$, for each semi-open set V of Y.

Proof. (1) ⟶ (2) Let V be any β-open subset of Y. It follows from Corollary 1.15, that clV is a regular closed set in Y. Since f is almost S_g-continuous, by Corollary 2.11, $f^{-1}(clV)$ is S_g-closed set in X. Therefore, we obtain $S_g clf^{-1}(V) \subseteq f^{-1}(clV)$.

(2) ⟶ (3) Let F be any β-closed set of Y. Then $Y \backslash F$ is a β-open set in Y and by (2), we have $S_g clf^{-1}(Y \backslash F) \subseteq f^{-1}(cl(Y \backslash F))$ and $S_g cl\ (X \backslash f^{-1}(F)) \subseteq (f^{-1}(Y \backslash intF))$, then by Corollary1.3, $X \backslash S_g int(f^{-1}(F)) \subseteq X \backslash f^{-1}(intF)$. Therefore, $f^{-1}(intF) \subseteq S_g int(f^{-11}(F))$.

(3)⟶ (4) Obvious since every semi-closed set is β-closed.

(4) ⟶ (5) Let V be any semi-open set in Y. Then $Y \backslash V$ is a semi-closed set in Y and by (4), we have $f^{-1}(intY \backslash V) \subseteq (S_g intf^{-1}(Y \backslash V))$ and $f^{-1}(Y \backslash clV) \subseteq S_g int\ (Y \backslash f^{-1}(V))$. Hence $X \backslash f^{-1}(clV) \subseteq X \backslash S_g cl(f^{-1}(V))$. Therefore, $S_g cl(f^{-1}(V)) \subseteq f^{-1}(clV)$.

(5) ⟶ (1) Let F be any regular closed set in Y. Then F is a semi-open set of Y. By (5), we have $S_g clf^{-1}(F) \subseteq f^{-1}(clF) = f^{-1}(F)$. This shows that $f^{-1}(F)$ is an S_g-closed set in X. Therefore, by Corollary 2.11, f is almost S_g-continuous.

Theorem 2.20 Let X be a locally indiscrete space. Then the function $f: (X, \tau) \to (Y, \sigma)$ is almost S_g-continuous if and only if $f: (X, \tau) \to (Y, \sigma_s)$ is continuous.

Proof. Necessity, let $H \in \sigma_s$. Then H is an δ-open set in (Y, σ). Since $f: (X, \tau) \to (Y, \sigma)$ is almost S_g-continuous, so by Proposition 2.12, $f^{-1}(H)$ is an S_g-open set in X. Since X is a locally indiscrete space, by Corollary1.12, $f^{-1}(H)$ is an open set in X. Therefore, $f: (X, \tau) \to (Y, \sigma_s)$ is continuous.

Sufficiency, let G be any δ-open set in (Y, σ). Then $G \in \sigma_s$. Since $f: (X, \tau) \to (Y, \sigma_s)$ is continuous, so $f^{-1}(G)$ is an open set in X. Since X is locally indiscrete, then by Corollary 1.12, $f^{-1}(G)$ is an S_g-open set in X. Therefore, by Proposition 2.12, $f: (X, \tau) \to (Y, \sigma)$ is almost S_g-continuous.

Proposition 2.21 A function $f: (X, \tau) \to (Y, \sigma)$ is almost S_g-continuous if and only if $f: (X, \tau) \to (Y, \sigma_s)$ is S_g-continuous.

Proof. It is obvious.

Theorem 2.22 If a function $f: X \to Y$ is δ-continuous and X is an extremally disconnected space, then f is almost S_g-continuous.

Proof. Let $x \in X$ and V be any open set in Y containing $f(x)$. Since f is δ-continuous, there exists an open set U of X containing x such that $f(int(cl(U))) \subseteq int(cl(V))$. Since $int(cl(U))$ is regular open and X is extremally disconnected, then by Theorem1.13, $int(clU))$ is regular closed and by Corollary1.9, $int(clU))$ is S_g-open. Hence it is an S_g-open set in X containing x. Therefore, f is almost S_g-continuous.

Theorem 2.23 If Y is a hyperconnected space, then every function $f: X \to Y$ is almost S_g-continuous.

Proof. Let Y be hyperconnected space and H be any open subset of Y containing $f(x)$. Then $clH = Y$. This implies that $intclH = Y$ and for every S_g-open set G in X containing x,$f(G) \subseteq Y = intclH$. Thus, f is almost S_g-continuous.

Theorem 2.24 Let (X, τ) be an s*- regular space and $f: (X, \tau) \to (Y, \sigma)$ is an almost S_g-continuous function, then f is almost α-continuous.

Proof. Let $x \in X$ and M be any open set of Y containing $f(x)$. Since f is almost S_g-continuous, there exists an S_g-open set U containing x such that $f(U) \subseteq intclM$. Since X is s*- regular, then by Theorem1.16, X is extremally disconnected and also U is semi-open set, then by Theorem1.13, U is α $-$open. So by Theorem1.17, f is almost α-continuous.

Theorem 2.25 Let $f: X \to Y$ be a function and X be a locally indiscrete space. If f is almost λ-continuous, then f is almost S_g-continuous.

Proof. Let V be any open set of Y containing $f(x)$. Since f is almost λ-continuous, then there exist a λ $-$open set U containing x such that $f(U) \subseteq intcl(V)$. Since X is locally indiscrete, then by Lemma 1.19, U is an open set in X, so U is semi-open in X. Again since X is locally indiscrete, then by Proposition 1.11, U is an S_g-open in X such that $f(U) \subseteq intcl(V)$. Hence f is almost S_g-continuous.

Theorem 2.26 If the graph function g of a function $f: X \to Y$ is almost S_g-continuous, then f is almost S_g-continuous.

Proof. Let g be almost S_g-continuous and $x \in X$ and U is any open subset of Y containing $f(x)$. Then $X \times U$ is an open set containing $g(x)$, so there exists an S_g-open set V in X such that $g(V) \subseteq intcl(X \times U) = X \times intclU$. This implies that $f(V) \subseteq intclU$. Hence f is almost S_g-continuous.

Lemma 2.27 Every almost S_g-continuous function is almost β-continuous.

Proof. Let $x \in X$ and let V be any regular open subset of Y containing $f(x)$. Since f is almost S_g-continuous, then by Theorem2.10, $f^{-1}(V)$ is an S_g-open set in X. This implies that $f^{-1}(V)$ is semi-open in X. Therefore, $f^{-1}(V)$ is β-open in X. Hence f is almost β-continuous.

Since every almost semi-continuous functions are almost β- continuous, then the identity function in Example 2.7 is almost S_g-continuous but not almost β-continuous. This means that the converse of Lemma 2.27 is not true in general.

Corollary 2.28 Let $f: X \to Y$ be a function and X is a completely regular (or regular) space. If f is almost continuous, then f is almost S_g-continuous.

Proof. Directly Follows from Proposition 1.10.

Proposition 2.29 A function $f: X \to Y$ is almost P_s-continuous (resp. almost precontinuous), if f is almost S_g- continuous and X is a locally indiscrete space.

Proof. Let V be any regular open subset of Y and f be almost S_g- continuous. Then by Theorem 2.10, $f^{-1}(V) \in S_gO(X)$. Since X is locally indiscrete, then by Corollary 1.3, $f^{-1}(V) \in \tau$. Again, since X is locally indiscrete, then by Theorem 1.18, $f^{-1}(V) \in P_sO(X)$. Hence f is almost P_s-continuous. by Corollary1.20, therefore f is almost precontinuous.

Lemma 2.30 Every almost strongly θ-continuous function is almost S_g-continuous.

Proof. Let V be any regular open set of Y containing $f(x)$. Since f is almost strongly θ-continuous, then by Theorem1.22, $f^{-1}(V)$ is a θ-open set in X and hence by Proposition1.23, $f^{-1}(V)$ is an S_g-open set. Therefore, by Theorem 2.10, f is almost S_g-continuous.

Since the function f in Example 2.5 is almost S_g-continuous and $\theta O(X) = \{\phi, X\}$, then f is not almost strongly θ-continuous. This implies that, the converse of Lemma 2.30 is not true in general.

Theorem 2.31 Let $f: X \to Y$ be a function and Y be a semi-regular space. Then f is almost $S_g$continuous if and only if f is S_g-continuous.

Proof. Let $x \in X$ and let V be any open subset of Y containing $f(x)$. By the semi-regularity of Y, there exists a regular open set G of Y such that $f(x) \in G \subseteq V$. Since f is almost S_g-continuous. By Theorem2.9, there exists an S_g-open set U in X containing x such that $f(U) \subseteq G \subseteq V$. Therefore, f is S_g-continuous.

The conversely, follows from Proposition 2.2.

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